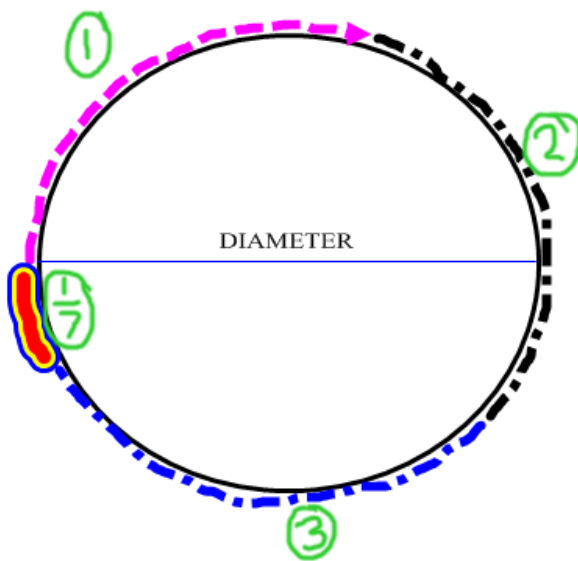


SINCE THE BEGINNING OF RECORDED TIME, MATHEMATICIANS AND BUILDERS HAVE WORKED TO UNDERSTAND THE RELATIONSHIP BETWEEN THE DIAMETER OF A CIRCLE AND ITS CIRCUMFERENCE.



each of the Mathletes worked hard to find the number of times the diameter can be stretched around circumference of circle the same way the Babylonians, Egyptians, and Chinese did.

They were more successful as they concluded that the multiple was about three and one seventh (very close to the value of π).

The Greek letter *pi* (written π) stands for the number by which the diameter of a circle (d) must be multiplied to obtain the circumference (c). That is , $c = \pi d$ or $2\pi r$, where r is the radius.

The area of a circle (A) is given by the formula $A = \pi r^2$

You cannot write π as a decimal. But by increasing the number of digits you can get a number as near to it as you want. Common values used for π include $22/7$, 3.14, 3.1416, and 3.141592653.

HISTORY: The ancient Chinese used 3 as the value of π . About 1650 B.C., the Egyptians improved on the approximation to 3 and $1/8$. The Greek mathematician Ptolemy calculated a value for π that was equivalent of 3.1416. After decimals came into use in the 1600's A. D., mathematicians worked hard to find an exact value in decimals for π . Mathematicians now know that this is impossible.

DURING THE LAST FOUR YEARS, I HAVE GIVEN
MY CELL PHONE NUMBER TO OVER 500
STUDENT MATHLETES:

617-817-0117

SINCE THE FIRST NINE DIGITS OF π ARE:

3.141592653

I HAVE REQUESTED THAT THEY CALL ME ON:

MARCH 14, 2015 @ 9:26 am and 53 seconds

3.141592653

"speak with you in eight years"

Mathletes learned how to use the graphing calculator by finding the symbol π .

when they pushed "*enter*," they found the decimal 3.141592654.

ROUNDING -- The ninth digit after the decimal on the calculator should be a 3, so why does it register "4"? First, look at the actual number π to 50 decimal places:
3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510.....
Since the digit after the 5 is a 3 and the next digit is a 5, the calculator, which runs out of digits at the ninth decimal, rounds the 3 up to 4. If the digit after the "3" were a 4 or less, the "3" would have stayed a "3." If the digit following the last digit to show is a 5 or greater, the calculator will round up to the next digit (in this case from 3 to 4).

When they compared $\pi = 3.141592653.....$

to $3 \frac{1}{7} = 3.142857$ (to understand how we got this decimal, see the next page)

The difference between the actual number π and the Mathletes' estimate of $3 \frac{1}{7}$ was so close; the difference is 12/100,000

*SO WE NOW UNDERSTAND THAT π
APPROXIMATELY IS :*

3 1/7 OR 22/7

mixed fraction -- it has a whole number followed by a proper fraction (numerator is smaller than its denominator)

improper fraction (no, this fraction didn't do anything wrong); it has a numerator which is greater than its denominator; therefore, it is larger than one.

CONVERTING MIXED FRACTIONS TO IMPROPER

- multiply the denominator of the proper fraction by the whole number to get the number of parts that make up the whole number (thus, 7×3 is 21 so $3 = 21/7$)
- then, we add the numerator of the proper fraction to obtain our improper fraction (thus, we add $21/7 + 1/7$ to get $22/7$)

CONVERTING IMPROPER FRACTIONS TO MIXED

- divide the numerator by the denominator to obtain your whole number ($22/7$ means 22 divided by 7, and we get 3 with a remainder of 1)
- the remainder is the number of parts (denominator) we have left (remainder of 1 gives us $1/7$ that we have to put next to the whole number 3, so $22/7 = 3 \frac{1}{7}$)

SO WE NOW WE ARE CURIOUS TO FIND OUT WHAT $3 \frac{1}{7}$ OR $\frac{22}{7}$ IS AS A DECIMAL SO WE CAN COMPARE IT TO π 's APPROXIMATION : 3.141592653

look at $3 \frac{1}{7}$, it already has the 3 -----> $3 \frac{1}{7}$

- so let's just convert $\frac{1}{7}$ to a decimal
- remember $\frac{1}{7}$ is the same as 1 divided by 7
- no, do not get out your calculator, here is where we use long division

$$\frac{1}{7} = 1 \div 7 = 7 \overline{)1}$$

when you get to a remainder that is the same as a previous remainder, the decimal will repeat forever

here is the **short division** approach to long division that all my Mathletes will be learning over the years.

$$\begin{array}{r} \overline{.142857} \\ 7 \overline{) 1.00000000} \end{array}$$

Handwritten long division of 1 by 7. The quotient is $.142857$ with a bar over it. The dividend is 1.00000000 . The first remainder is 1, and the eighth remainder is also 1, with a dashed pink arrow connecting them to show the repeating pattern. Green numbers 3, 2, 6, 4, 5, 1 are written above the zeros.

- when you are working with endless zeros, once you see a remainder like "1" that is a repeat of the first remainder of "1" you know that the pattern of answers will repeat infinitely.
- we put a bar over the repeat to indicate that it goes on indefinitely.
- so $1/7 = .142857\ 142857\ 142857\ 142857\ 142857\ \dots\dots\dots$

WE CAN TAKE THE DIAMETER OF ANY CIRCLE AND MULTIPLY IT BY π AND GET THE CIRCUMFERENCE OF THAT CIRCLE.

SO, THE RATIO OF THE CIRCUMFERENCE AND DIAMETER OF A CIRCLE IS π .

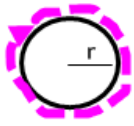
$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

SOME OF THE MATHLETES USED THE GRAPHING CALCULATOR TO CREATE TABLES OF DIAMETERS WITH RESULTING CIRCUMFERENCES:

<u>diameter</u>	<i>linear</i> <u>circumference</u>	<i>square</i> <u>area</u> ($\pi \times (1/2 \text{ diameter})^2$)
1	3.14	3.14
2	6.28	12.57
3	9.42	28.27
4	12.57	50.27
5	15.71	78.54
6	18.85	113.1
7	22.00	153.94
8	25.13	201.06
9	28.27	254.47
10	31.42	314.16

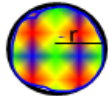
memorizing digits of π is cool, but it is even more important for Mathletes to understand the many uses of π , including:

r = radius H = height of the cylinder



CIRCUMFERENCE OF A CIRCLE (perimeter)

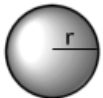
$$2\pi r$$



AREA OF A CIRCLE

(inside square measurement)

$$\pi r^2$$



VOLUME OF A SPHERE

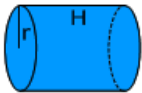
(solid inside, cubic measurement)

$$\frac{4}{3} \pi r^3$$



SURFACE AREA OF A SPHERE (outside square measurement)

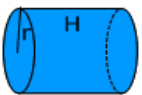
$$4\pi r^2$$



VOLUME OF A CYLINDER

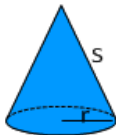
(solid inside, cubic measurement)

$$\pi r^2 H$$



SURFACE AREA OF CYLINDER (outside square measurement)

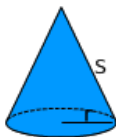
$$2\pi r^2 + 2\pi r H$$



VOLUME OF A CONE

(solid inside, cubic measurement)

$$\frac{1}{3} \pi r^2 H$$



SURFACE AREA OF CONE

(outside square measurement)

$$\pi r^2 + \pi r S$$

S = SLANT HEIGHT = DISTANCE FROM ANY EDGE OF THE BASE TO THE APEX (POINT)